Part 12: Oscillations, Waves, and Sound

Physics for Engineers & Scientists (Giancoli): Chapters 11 & 12
University Physics V1 (Openstax): Chapters 15, 16 & 17

Simple Harmonic Motion

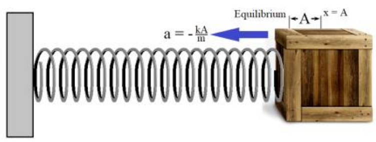
- Simple Harmonic Motion is a repetitive (periodic) state of motion that occurs when the magnitude of the restoring force is proportional to the displacement from equilibrium.

One place this type of behavior occurs is when a mass is attached to a spring and allowed to slide across a frictionless surface.

- If we pull the mass a distance A away from equilibrium, the Restoring Force of the spring will pull it back towards the equilibrium position.

$t = 0$

$t = nT$



$x = x_{max} = A$
$v = 0$
$a = -a_{max} = -\frac{kA}{m} = -\omega^2 A$

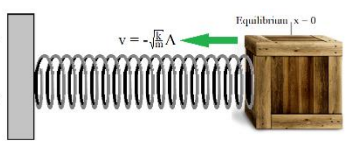
  - The Amplitude (A) is the maximum displacement of the system from equilibrium.
  - The mass is released from rest in this case. $v_0 = 0$.
  - The acceleration is: $a = \frac{F}{m} = \frac{-kx}{m} = \frac{-kA}{m}$
  - As will be shown later, the angular frequency (ω) of the system is: $\omega = \sqrt{\frac{k}{m}}$
  - The object will return to this exact state of the beginning of every period

$$t = nT, \quad for\ n = 0, 1, 2, \ldots$$

- After a quarter of the period, the mass will have returned to its equilibrium position.

$t = \frac{1}{4}T$

$t = (n + \frac{1}{4})T$



$x = 0$
$v = -v_{max} = -\sqrt{\frac{k}{m}}A = -\omega A$
$a = 0$

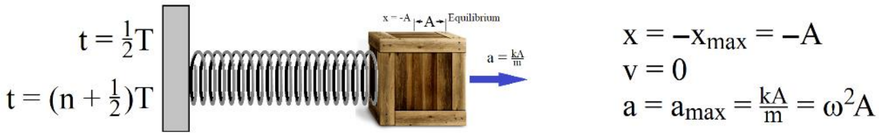
  - At equilibrium ($x = 0$) the net force is zero. This means there is no longer any acceleration.
  - One it passes equilibrium it will begin to decelerate, making this the maximum speed.
  - We can calculate that speed using conservation of energy:

$$E_{Init} = U_{Elastic} = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \qquad E_{Final} = KE = \frac{1}{2}mv^2$$

$$E_{Init} = E_{Final} \qquad \frac{1}{2}kA^2 = \frac{1}{2}mv^2 \qquad kA^2 = mv^2 \qquad \frac{k}{m}A^2 = v^2 \qquad v = \sqrt{\frac{k}{m}}A = \omega A$$

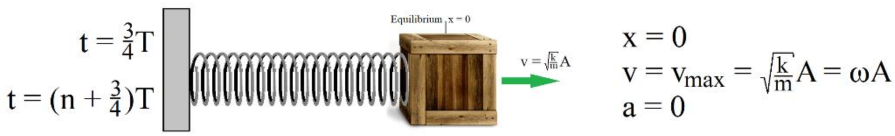
  - The object will return to this state every period: $t = (n + \frac{1}{4})T, \quad for\ n = 0, 1, 2, \ldots$

- After half of the period, the mass will have come to rest again.



- The amplitude has maximum magnitude again, this time on the negative side: x = -A.
- The mass has come to rest again. $v_0 = 0$.
- The acceleration is: $a = \frac{F}{m} = \frac{-kx}{m} = \frac{kA}{m}$
- The object will return to this state every period: $t = (n + \frac{1}{2})T, \quad for\ n = 0, 1, 2, \ldots$

- After three quarters of the period, the mass will have returned to its equilibrium position again.

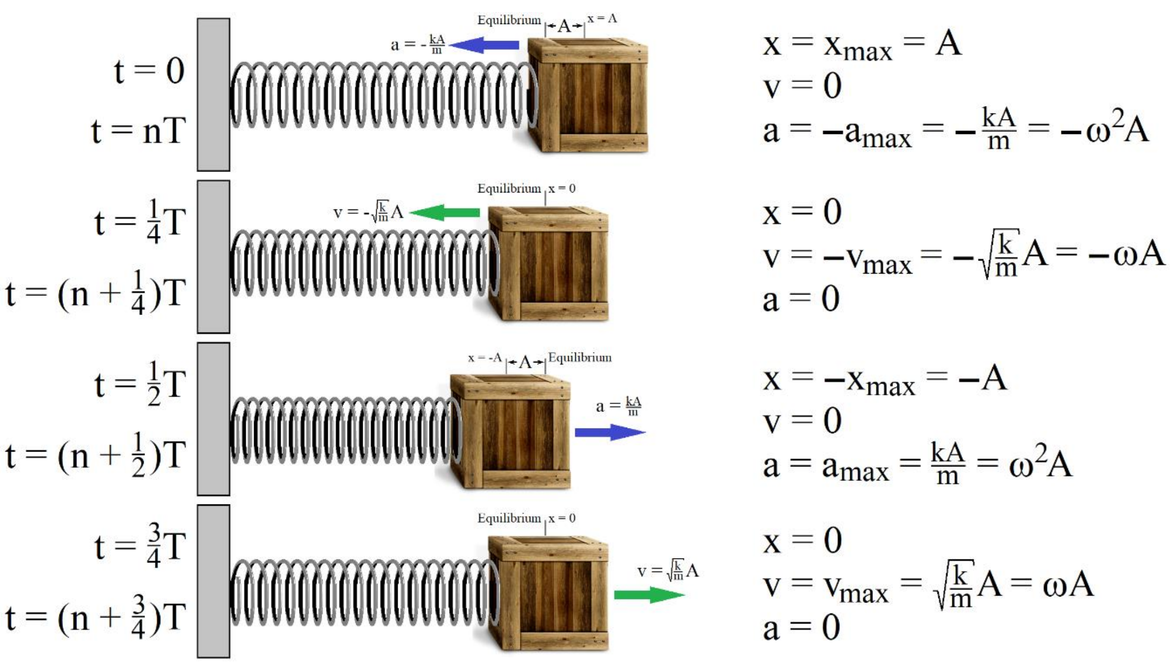


- At equilibrium (x = 0) the net force is zero. This means there is no longer any acceleration.
- One it passes equilibrium it will begin to decelerate, making this the maximum speed.

$$v = \sqrt{\frac{k}{m}}A = \omega A$$

- The object will return to this state every period: $t = (n + \frac{3}{4})T, \quad for\ n = 0, 1, 2, \ldots$

- After one full period, the object returns to its initial position and state and the cycle begins again.



- The energy of the system continually changes from potential energy to kinetic energy and back.
- The velocity at any position can be calculated using conservation of energy:

$$E_{system} = E_{Init} = \frac{1}{2}kA^2 \qquad E_{Final} = U_{Elastic} + KE = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$E_{Init} = E_{Final} \qquad \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}mv^2 \qquad k(A^2 - x^2) = mv^2 \qquad \frac{k}{m}(A^2 - x^2) = v^2 \qquad v = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

- General Solution

$$F = ma \qquad -kx = m\frac{d^2x}{dt^2} \qquad m\frac{d^2x}{dt^2} + kx = 0 \qquad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

The solution to this well-known differential equation is of the form: $x(t) = C_1 \cos\omega t + C_2 \sin\omega t$

The given initial position at t=0 is x=A. $x(0) = C_1 \cos 0° + C_2 \sin 0° = C_1 = A$

The given initial position is a maximum. $C_2 = 0$

$$x(t) = A\cos\omega t \qquad \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(A\cos\omega t) = \frac{d}{dt}(-\omega A\sin\omega t) = -\omega^2 A\cos\omega t$$

Plug into the initial different equation:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad -\omega^2 A\cos\omega t + \frac{k}{m}A\cos\omega t = 0 \qquad \left(\frac{k}{m} - \omega^2\right)A\cos\omega t = 0$$

As A ≠ 0 and cos(ωt) ≠ 0 (at least it isn't for all values of t):

$$\frac{k}{m} - \omega^2 = 0 \qquad \omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Which gives us our solution: $x(t) = A\cos\omega t$ *with* $\omega = \sqrt{\frac{k}{m}}$

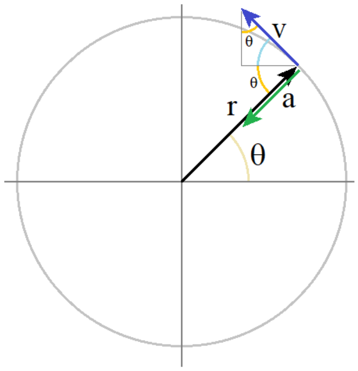
$$v = \frac{dx}{dt} = \frac{d}{dt}(A\cos\omega t) = -\omega A\sin\omega t \qquad v_{max} = \omega A = \sqrt{\frac{k}{m}}A$$

$$a = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(A\cos\omega t) = -\omega^2 A\cos\omega t \qquad a_{max} = \omega^2 A = \frac{k}{m}A$$

This solution is only valid when the initial position is a maximum.

Simple Harmonic Motion vs. Uniform Circular Motion

- The x- and y-components of uniform circular motion are in simple harmonic motion.
  - Simple harmonic motion can be viewed as a 1-dimensional view of uniform circular motion.
  - Uniform circular motion can be viewed as 2-dimensional simple harmonic motion where the two components are 90° out of phase.



Let the radius of the circle be the amplitude (A). $r = A$

There is no angular acceleration. $\theta = \omega t + \theta_0$

$x = r\cos\theta = A\cos(\omega t + \theta_0)$

$v_x = -v\sin\theta = -\omega r\sin(\omega t + \theta_0) = -\omega A\sin(\omega t + \theta_0)$

$a_x = -a\cos\theta = -\frac{v^2}{r}\cos(\omega t + \theta_0) = -\frac{(\omega r)^2}{r}\cos(\omega t + \theta_0)$

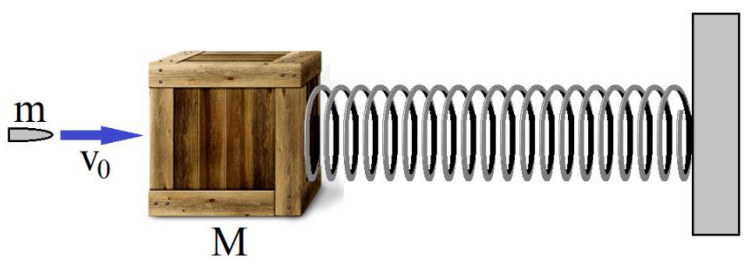
$a_x = -\omega^2 r\cos(\omega t + \theta_0) = -\omega^2 A\cos(\omega t + \theta_0)$

Example: Military aircraft and pilots are tested to ensure they can withstand accelerations of 9g (88.2 m/s^2). To ensure that communication equipment can withstand these g-forces it is placed on an oscillating table that shifts back and forth in simple harmonic motion at a frequency of 5.25 Hz. To ensure that the equipment is tested at a maximum acceleration of 9g, what amplitude is needed?

$$a_{max} = \omega^2 A = (2\pi f)^2 A = 4\pi^2 f^2 A = 9g$$

$$A = \frac{9g}{4\pi^2 f^2} = \frac{9\left(9.80\,\frac{m}{s^2}\right)}{4\pi^2(5.25\ Hz)^2} = 8.11\ cm$$

Example: A 50.0 kg block is attached to a spring (k = 450 N/m), which in turn is attached to a wall. The block is at rest when it is struck by a bullet with a trajectory that would pass straight down the center of the spring. The bullet becomes lodged in the block, and sends it into simple harmonic motion with frequency of 0.4765 Hz and amplitude 50.5 cm. Determine the mass and the initial velocity of the bullet.



$$\omega = \sqrt{\frac{k}{m+M}} = 2\pi f \qquad \frac{k}{m+M} = 4\pi^2 f^2 \qquad m + M = \frac{k}{4\pi^2 f^2}$$

$$m = \frac{k}{4\pi^2 f^2} - M = \frac{\left(450\,\frac{N}{m}\right)}{4\pi^2(0.4765)^2} - 50.0\ kg = 0.20269\ kg$$

Conservation of momentum relates the bullet velocity (v_0) to V.

$$mv_0 = (m + M)V \qquad v_0 = \left(1 + \frac{M}{m}\right)V$$

Conservation of energy (after the collision) relates V to A.

Or…

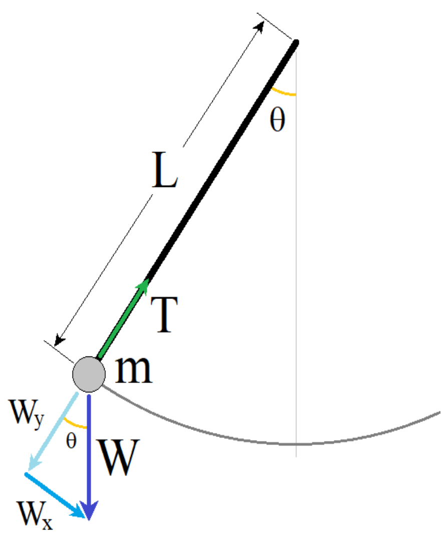
The velocity of the block and bullet (V) right after the collision occurs at equilibrium. That means V is the maximum velocity.

$$V = v_{max} = \omega A = 2\pi f A$$

$$v_0 = \left(1 + \frac{M}{m}\right)V = \left(1 + \frac{M}{m}\right)2\pi f A = \left(1 + \frac{50.0\ kg}{0.20269\ kg}\right)2\pi(0.4765\ \text{Hz})(0.505\ m) = 374\frac{m}{s}$$

Pendulum

- We will treat this as 1-dimensional simple harmonic motion along the arc made by the hanging mass.



Weight (W) and Tension (T) act on the hanging mass.

One component of the weight (W_y) cancels out the tension.

The other component of the weight (W_x) acts as the restoring force.

If we can find k, then we can use $\omega = \sqrt{\frac{k}{m}}$

$$k = -\frac{F}{x} = -\frac{W_x}{-L\theta} = \frac{mg\sin\theta}{L\theta} \approx \frac{mg\theta}{L\theta} = \frac{mg}{L}$$

In the small angle approximation, $\sin\theta \approx \theta$

$$\omega = \sqrt{\frac{mg}{Lm}} = \sqrt{\frac{g}{L}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

- The period/frequency of a pendulum is independent of mass.

Example: The pendulum in Big Ben has a 299 kg bob and a period of 2 seconds. What is the length of the arm of this pendulum?

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \frac{T}{2\pi} = \sqrt{\frac{L}{g}} \qquad \frac{T^2}{4\pi^2} = \frac{L}{g} \qquad L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80\frac{m}{s^2}\right)(2.00\ s)^2}{4\pi^2} = 99.3\ cm$$

Example: Wilson Hall, the picturesque administrative building at Fermilab, used to have a rather slow-moving pendulum hanging from the very top of the building, 16 floors high (roughly 160 ft.). How long does it take this pendulum to make one complete cycle?

$$L = (160\ ft.)\frac{(0.3048\ m)}{(1\ ft.)} = 48.768\ m$$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{48.768\ m}{9.80\frac{m}{s^2}}} = 14.0\ s$$

Damping, Driving, and Resonance

- Springs are useful for absorbing impacts.

- Choosing the spring constant allows smaller deceleration to occur over a larger distance, reducing the force of impact.

- Springs absorb and release a portion of the impact energy, which can lead to unwanted harmonic motion.

- In **Damped Harmonic Motion**, energy is steadily removed from the system resulting in decreasing amplitude.

Cars are suspended on springs so that when you drive over a bump the deceleration is gentler. If the absorbed collision energy is not dissipated by damping (shock absorbers), then your vehicle would continue to bounce.

- In **Driven Harmonic Motion**, energy is added from an outside source.
  - How a driven harmonic oscillator behaves is dependent upon both the frequency of the driving force and the natural frequency of the oscillator.
  - **Resonance** occurs when the driving frequency and the oscillator frequency match. When this happen, energy is continually added to the system.

When pushing a child in a swing, timing your pushes to the timing of the swing results in the child swinging higher and higher (ever increasing amplitude) even as the friction in the system (damping) causes their swinging to slow down and lose height.

The Wave Equation and its Solutions

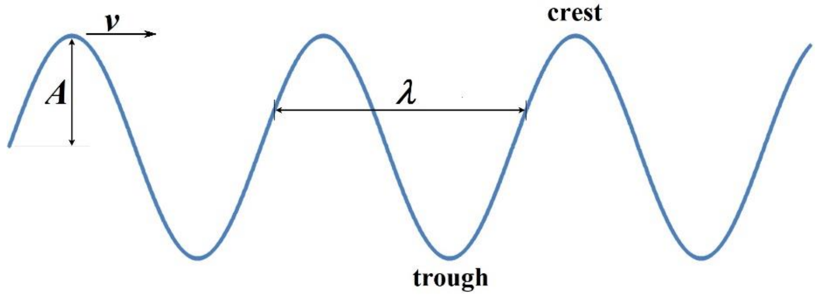
- In some instances, Newton's laws lead to a partial differential equation known as the Wave Equation.

$$\frac{\partial^2 D}{\partial t^2} - v^2\frac{\partial^2 D}{\partial x^2} = 0$$

D = Displacement as measured from equilibrium.
The constant (v^2) is the square of the velocity of the wave.

- The solutions to this equation are travelling waves, either a sine function, a cosine function, or a combination of the two depending upon the initial phase.

$$D = A\sin(kx - \omega t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$



- The Wave Number (k) is the spatial frequency of the wave (cycles per unit distance).

$$k = \frac{2\pi}{\lambda} \qquad v = \frac{\omega}{k} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda = \frac{\lambda}{T}$$

- The velocity (v) is determined by the properties of the medium through which the wave moves.
- The frequency (f), angular frequency (ω), period (T), wavelength (λ), and wave number (k) are all inter-related and determined by whatever excitation created the wave.

- Travelling waves obey **Superposition**, meaning that the displacements of two different waves simply add together, superimposing one wave on top of the other.
- When waves of the same frequency combine (superimpose), it is called **Interference** and can create a resultant wave of greater (**Constructive Interference**) or lower (**Destructive Interference**) amplitude.
- Driven travelling waves may also experience **Resonance**.
- Travelling waves are **Longitudinal** if the movement of particles making the wave is parallel/anti-parallel with the direction of the wave's motion.
- Travelling waves are **Transverse** if the movement of particles making the wave is perpendicular to the direction of the wave's motion.
- Travelling waves often reflect back when they encounter boundaries and may invert (180° phase shift) upon reflection.

Example: The amplitude of an ocean swell is 1.50 m with crests separated 33.8 m. A wave crest strikes the beach once every 5.70 s. Determine (A) the frequency of the waves, (B) the speed of the waves, and (C) the wave number.

A = 1.50 m $\qquad$ λ = 33.8 m $\qquad$ T = 5.70 s.

$$f = \frac{1}{T} = \frac{1}{5.70s} = 0.175\,Hz \qquad v = \frac{\lambda}{T} = \frac{33.8\,m}{5.70\,s} = 5.93\,\frac{m}{s} \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi}{33.8\,m} = 0.186\,m^{-1}$$

Transverse Waves on Strings

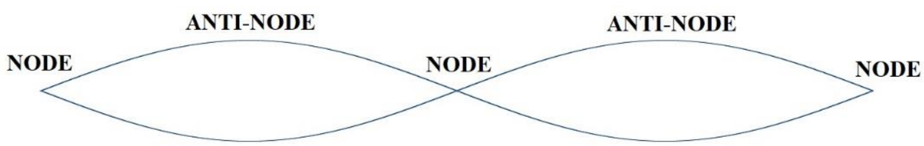
- The velocity of waves on a string are given by: $\quad v = \sqrt{\frac{F_T}{\mu}}$

- F_T is the tension in the string (so as not to be confused with the period T).
- μ is the mass per unit length of the string.

The length of a string determines the wavelengths allowed. If we assume λ is fixed, then the frequency of the wave increases with the velocity. $f = v/\lambda$.

- Increasing the tension (typically by turning a tuning nut) increases the velocity of the waves, creating higher pitched notes.
- Thicker strings have higher values of μ, and higher values of μ lead to lower velocities and lower pitched notes.

- Waves travelling down a string will reflect back from the ends of the string interfering with the original waves.
  - The majority of frequencies experience destructive interference.
  - Only a few specific modes of vibration (also known as **Harmonics**) experience constructive interference. In these cases, the superposition of the original wave and its reflections results in a **Standing Wave**, a wave that oscillates in time at amplitudes that are fixed in space.



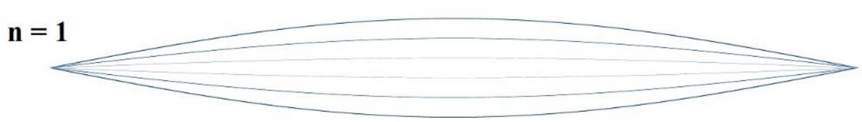
  - In each mode, certain positions called on the string **Nodes** have zero amplitude (no vibration). As the ends of strings are typically held in place (as the string is under tension), these must become nodes.
  - Certain other positions called **Anti-Nodes** are vibrating with the maximum amplitude.

All modes begin vibrating when a string is plucked. Normally the amplitudes of these various modes fall as you move to higher harmonics (making the higher modes significantly quieter). The various amplitudes are also affected by how and where a string is struck or plucked. The sum total of all of these modes produces the sound that you hear.

  - Each mode of vibration (n) occurs at a specific wavelength (and frequency) related to the string's length (L).

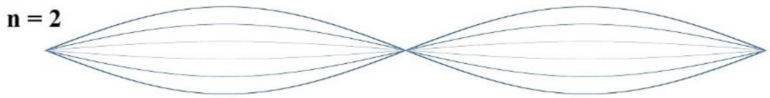
$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{v_0}{\lambda_n} = \frac{nv_0}{2L} \qquad \#Nodes = n + 1 \qquad \#Anti\text{-}Nodes = n$$

    - The first harmonic or fundamental mode (n = 1) is the simplest and usually the loudest tone heard (largest amplitude). It has a node at each end, and one anti-node in between.



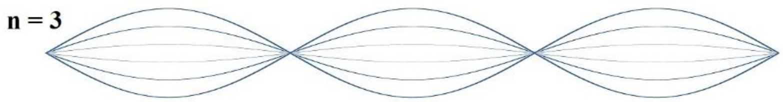
$$L = \frac{1}{2}\lambda_1 \qquad \lambda_1 = 2L \qquad f_1 = \frac{v_0}{\lambda_1} = \frac{v_0}{2L}$$

- The second harmonic (n = 2) has a node at each end, another node in the middle, and two anti-nodes (halfway between each adjacent pair of nodes).



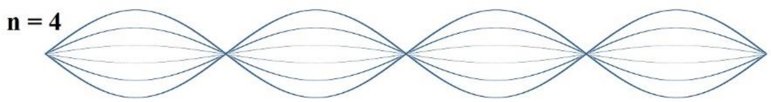
$$\lambda_2 = L \qquad f_2 = \frac{v_0}{\lambda_2} = \frac{v_0}{L}$$

- The third harmonic (n = 3) has a node at each end, two more nodes evenly spaced in between, and three anti-nodes (halfway between each adjacent pair of nodes).



$$L = \frac{3}{2}\lambda_3 \qquad \lambda_3 = \frac{2L}{3} \qquad f_3 = \frac{v_0}{\lambda_3} = \frac{3v_0}{2L}$$

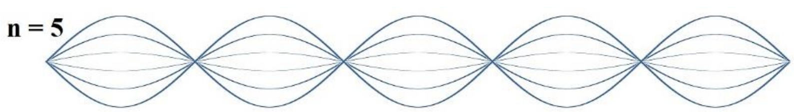
- The fourth harmonic (n = 4) has 5 nodes and 4 anti-nodes.



$$L = 2\lambda_4 \qquad \lambda_4 = \frac{L}{2} \qquad f_4 = \frac{v_0}{\lambda_4} = \frac{2v_0}{L}$$

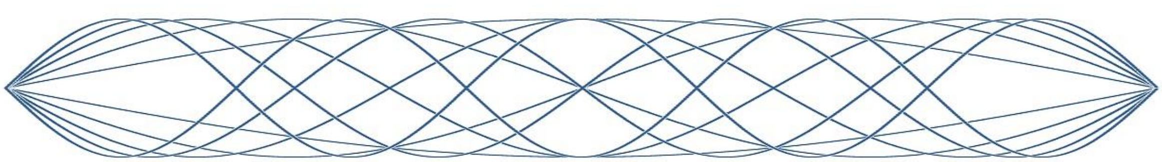
Sometimes a guitarist will play a "harmonic" by holding their finger gently on the string without pressing it against the fret board (often to tune). This forces a node at that position, suppressing all modes of vibration that don't have this node (including the fundamental). If done at the 5th fret sounds are dominated by the 4th harmonic mode. The 7th fret gives you the 3rd harmonic mode, and the 12th fret gives you the second.

- The fifth harmonic (n = 5) has 6 nodes and 5 anti-nodes.



$$L = \frac{5}{2}\lambda_5 \qquad \lambda_5 = \frac{2L}{5} \qquad f_5 = \frac{v_0}{\lambda_5} = \frac{5v_0}{2L}$$

When the envelopes of the first five harmonics of a string are superimposed, it makes a rather striking image.

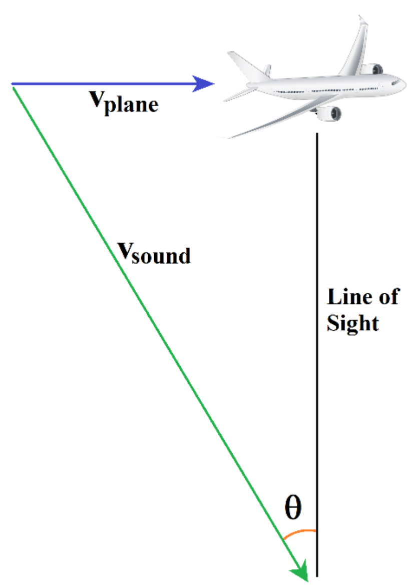


Sound Waves

- Sound is a pressure wave (also called a compression wave).
- The velocity of sound waves in liquid or gas: $v_{sound} = \sqrt{\frac{\beta}{\rho}}$
    - β is the bulk modulus and ρ is the mass density of the medium.
    - The speed of sound in air varies with temperature. At 20°C: $v_{sound} = 343\frac{m}{s}$
    - The speed of sound in water depends on pressure (depth), temperature, and salinity. On average the speed of sound is 1560 m/s in saltwater and 1435 m/s in freshwater.
    - In liquids and gases, sound is strictly a longitudinal wave with alternating bands of high pressure (crests) and low pressure (troughs).
- The velocity of sound waves in a solid: $v_{sound} = \sqrt{\frac{Y}{\rho}}$
    - Y is the Young's modulus and ρ is the mass density of the medium.
    - The speed of sound in iron is roughly 5130 m/s.
    - In solids sound can propagate as either a longitudinal or a transverse wave. The transverse waves are alternating shear stress at a right angle to the propagation, and the longitudinal waves are alternating bands of high pressure (crests) and low pressure (troughs).

Seismic activity (earthquakes) generates both (primary) longitudinal waves (P-waves) and (secondary) transverse waves (S-waves). The longitudinal waves travel at roughly 5000 m/s (in granite) while the slower transverse waves travel at only 3000 m/s. Typically the transverse waves do greater damage as they are typically created with larger amplitudes.

Example: As a plane flies overhead you notice that the sound of the engines appears to be coming from a spot 20.0° behind the aircraft. How fast is the airplane moving?



$$v_{airplane} = v_{sound}\sin\theta = \left(343\frac{m}{s}\right)\sin 20.0° = 117\frac{m}{s}$$

117 m/s is about 262 mph.
At altitudes below 10,000 ft., aircraft are limited to a maximum speed of 250 knots (288 mph).

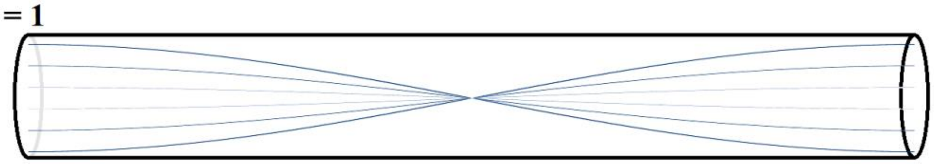
Longitudinal Waves in a Pipe

- A closed end of a pipe creates a node while an open end creates an anti-node.
- Each mode of vibration (n) occurs at a specific wavelength (and frequency) related to the pipe's length (L), depending upon whether one end or both ends are open.
- Harmonics in pipe with two open ends:

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{v_{sound}}{\lambda_n} = \frac{n v_{sound}}{2L} \qquad \#Nodes = n \qquad \#Anti\text{-}Nodes = n + 1$$

  - The first harmonic or fundamental mode (n = 1) is the simplest and usually the loudest tone heard (largest amplitude). It has an anti-node at each end, and one node in between.

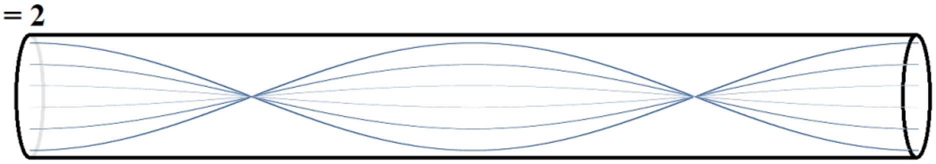
n = 1



$$L = \frac{1}{2}\lambda_1 \qquad \lambda_1 = 2L \qquad f_1 = \frac{v_{sound}}{\lambda_1} = \frac{v_{sound}}{2L}$$

  - The second harmonic (n = 2) has an anti-node at each end, another anti-node in the middle, and two nodes (halfway between each adjacent pair of anti-nodes).

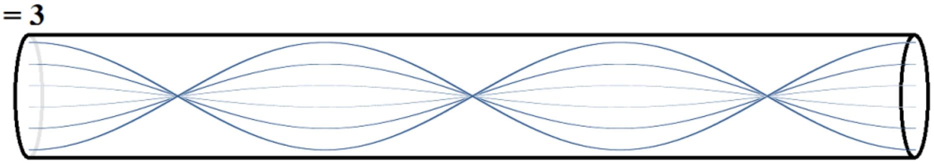
n = 2



$$\lambda_2 = L \qquad f_2 = \frac{v_{sound}}{\lambda_2} = \frac{v_{sound}}{L}$$

  - The third harmonic (n = 3) has an anti-node at each end, two more anti-nodes evenly spaced in between, and three nodes (halfway between each adjacent pair of anti-nodes).

n = 3



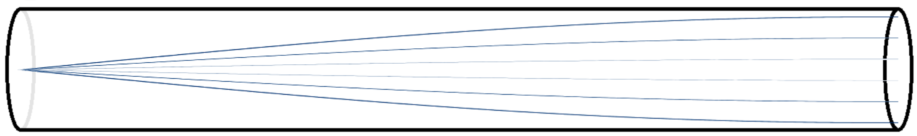
$$L = \frac{3}{2}\lambda_3 \qquad \lambda_3 = \frac{2L}{3} \qquad f_3 = \frac{v_{sound}}{\lambda_3} = \frac{3v_{sound}}{2L}$$

- Harmonics in pipe with one end open and one end closed:

$$\lambda_n = \frac{4L}{2n-1} \qquad f_n = \frac{v_{sound}}{\lambda_n} = \frac{(2n-1)v_{sound}}{4L} \qquad \#Nodes = n \qquad \#Anti\text{-}Nodes = n$$

  - The first harmonic or fundamental mode (n = 1) is the simplest and usually the loudest tone heard (largest amplitude). It has a node at one end and an anti-node at the other.

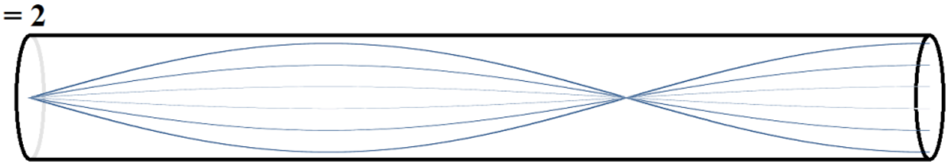
n = 1



$$L = \frac{1}{4}\lambda_1 \qquad \lambda_1 = 4L \qquad f_1 = \frac{v_{sound}}{\lambda_1} = \frac{v_{sound}}{4L}$$

  - The second harmonic (n = 2) has a node and an anti-nodes at either end, with another node and anti-node in the middle.

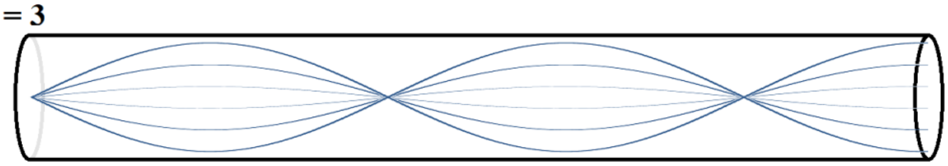
n = 2



$$L = \frac{3}{4}\lambda_2 \qquad \lambda_2 = \frac{4}{3}L \qquad f_2 = \frac{v_{sound}}{\lambda_2} = \frac{3v_{sound}}{4L}$$

  - The third harmonic (n = 3) has three nodes and three anti-nodes.

n = 3



$$L = \frac{5}{4}\lambda_3 \qquad \lambda_3 = \frac{4L}{5} \qquad f_3 = \frac{v_{sound}}{\lambda_3} = \frac{5v_{sound}}{4L}$$

Sometimes musicians playing horns will cover the opening, providing a drop in frequency (one octave) and a change in the harmonic frequencies.

Example: Two pipes are the same length. The first pipe has both ends open, and the second has one closed end. If the both pipes have their 4th harmonic excited, which pipe produces the higher pitch?

1st Pipe: $f_n = \frac{nv_{sound}}{2L} = \frac{4v_{sound}}{2L} = 2\frac{v_{sound}}{L}$

2nd Pipe: $f_n = \frac{(2n-1)v_{sound}}{4L} = \frac{[2(4)-1]v_{sound}}{4L} = \frac{7}{4}\frac{v_{sound}}{L}$

As $2 > \frac{7}{4}$, the 1st pipe produces the higher pitch.

<u>Intensity</u> (I): $I = \frac{P}{A}$

- The **<u>Intensity</u> (I)** of a wave is the power per unit area carried by the wave. Units: $\frac{W}{m^2}$
- Typically a time averaged value is used for power.
- As sound tends to radiate spherically, the intensity will drop with the square of the radius.

<u>Sound Levels</u> (β) $\beta(dB) = 10log\left(\frac{I}{I_0}\right)$

- Normal human hearing is sensitive to frequencies from 20 Hz to 20 kHz, but is most sensitive to sounds between 1 kHz and 4 kHz.
- The minimum intensity that we can hear is $I_0 = 10^{-12}\,\frac{W}{m^2}$
- As human hearing is sensitive to a wide range of intensity, a log scale is used for sound levels.

$$\beta(dB) = 10log\left(\frac{I}{I_0}\right)$$

- Sound below 75 dB typically does no damage to hearing.
  - Breathing (10 dB) is barely audible.
  - Whispering or a Quiet Rural Area (30 dB) is very quiet.
  - Conversation in restaurant, office background music, or an air conditioner at 100 ft. (60 dB) is fairly quiet.
  - Vacuum Cleaner (70 dB)
- Intense sounds (85 dB and above) can damage a person's ability to hear depending upon the duration of exposure.
  - 8 hours of exposure to sounds over 90 dB can possibly cause damage. This includes the sound of a power mower (96 dB), being 25 ft. from a motorcycle (90 dB), or being 1 nautical mile (6080 ft.) from a landing commercial aircraft (97 dB)
  - Sounds at the average human pain threshold of 110 dB such as a car horn at 1m (110 dB) or live music at a rock concert (108 to 114 dB) can cause damage in minutes.
  - Sounds at 120dB (such as a chainsaw) are painful and can damage an ear after only seconds.
  - Sounds at 150 dB (such a jet taking off at 25 meters) can rupture eardrums.

<u>Example</u>: The sound technician in a recording studio sets the sound level of the backing vocals to be 3.00 dB lower than the lead vocals. Determine the ratio of the intensity of the background vocals to that of the lead vocals.

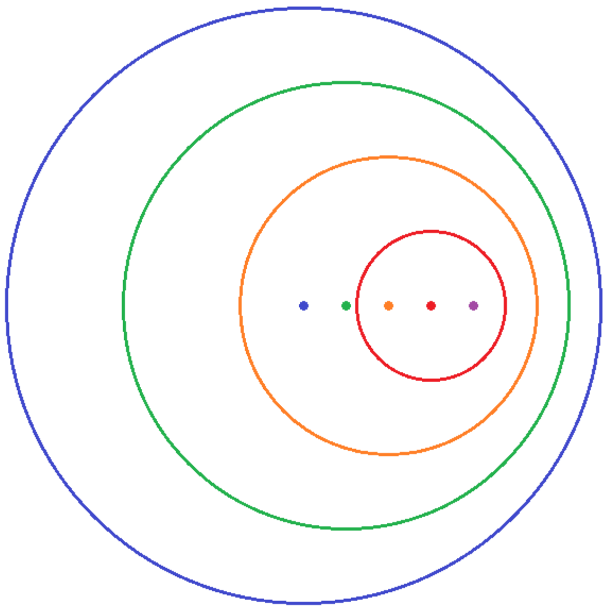
$$\beta_B = \beta_L - 3.00 \qquad 10log\left(\frac{I_B}{I_0}\right) = 10log\left(\frac{I_L}{I_0}\right) - 3.00 \qquad log\left(\frac{I_B}{I_0}\right) = log\left(\frac{I_L}{I_0}\right) - 0.300$$

$$10^{log\left(\frac{I_B}{I_0}\right)} = 10^{log\left(\frac{I_L}{I_0}\right)-0.300} = 10^{log\left(\frac{I_L}{I_0}\right)}10^{-0.300} = \frac{I_L}{I_0}(0.501)$$

$$\frac{I_B}{I_0} = 0.501\frac{I_L}{I_0} \qquad \frac{I_B}{I_L} = 0.501$$

The Doppler Effect

- Moving sources of sound compress the wavelengths in front of them and stretch the wavelengths behind them. A stationary observer will hear a different frequency than that emitted by the source.
- A similar effect occurs when the observer is moving.



$$f' = f\left(\frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}\right)$$

f' is the frequency heard by the observer.

f is the frequency emitted by the source.

v_{sound} is the speed of sound in air (343 m/s).

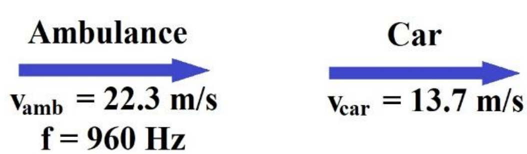
$v_{observer}$ is the velocity of the observer.

v_{source} is the velocity of the source.

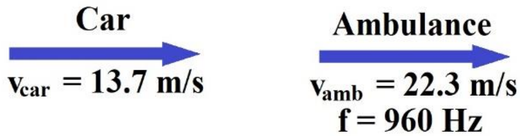
The top sign is used when the velocity of the observer (source) is directed towards the source (observer).

Or, if you prefer, if you always place the source on the right and the observer on the left, then us a positive sign when moving right and a negative sign when moving left.

Example: A car is moving down a street at 13.7 m/s when the driver hears the siren of an ambulance approaching from behind at 22.3 m/s. The frequency of the horn on the ambulance is 960 Hz. (A) What frequency does the driver of the car hear as the ambulance approaches? And (B) What frequency does the driver of the car hear after being passed by the ambulance?



$$f' = f\left(\frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}\right) = (960\ Hz)\left(\frac{343\frac{m}{s} - 13.7\frac{m}{s}}{343\frac{m}{s} - 22.3\frac{m}{s}}\right) = 986\ Hz$$



$$f' = f\left(\frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}\right) = (960\ Hz)\left(\frac{343\frac{m}{s} + 13.7\frac{m}{s}}{343\frac{m}{s} + 22.3\frac{m}{s}}\right) = 937\ Hz$$

Example: A bat is flying at 4.50 m/s towards an insect the bat intends to feed upon. The insect is moving towards the bat at 3.00 m/s. The bat chirps at a frequency of 100 kHz. Determine the frequency of the reflected sound heard by the bat.

Let f' be the frequency observed by the insect. As this is what reflects, it becomes the source for the return trip. We shall call f'' the frequency heard by the bat.

$$f' = f\left(\frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}\right) = f\left(\frac{v_{sound} + v_{insect}}{v_{sound} - v_{bat}}\right)$$

$$f'' = f'\left(\frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}\right) = f'\left(\frac{v_{sound} + v_{bat}}{v_{sound} - v_{insect}}\right) = f\left(\frac{v_{sound} + v_{insect}}{v_{sound} - v_{bat}}\right)\left(\frac{v_{sound} + v_{bat}}{v_{sound} - v_{insect}}\right)$$

$$f'' = (100\ kHz)\left(\frac{343\frac{m}{s} + 3.00\frac{m}{s}}{343\frac{m}{s} - 4.50\frac{m}{s}}\right)\left(\frac{343\frac{m}{s} + 4.50\frac{m}{s}}{343\frac{m}{s} - 3.00\frac{m}{s}}\right) = 104\ kHz$$